TALLER DEL CENTRO DE APRENDIZAJE DE MATEMÁTICAS

Demostraciones de *Límites*

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- **1. LIMITES EN UN PUNTO**

$$\lim_{x \to a} f(x) = L$$

Para todo $\epsilon > 0$ existe $\delta > 0$ tal que $|f(x) - L| < \epsilon$ si $0 < |x - a| < \delta$

EJEMPLO 1. Demostrar :

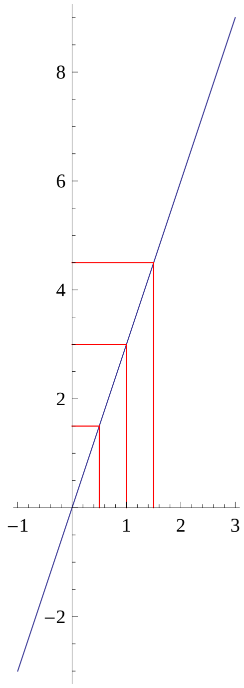
$$\lim_{x \to 1} f(x) = 3$$

$$f(x) = 3x$$

Sea $\epsilon > 0$, encontremos $\delta > 0$ tal que $|3x - 3| < \epsilon$ si $0 < |x - 1| < \delta$.

$$|3x - 3| = |3(x - 1)| = |3|\,|x - 1| = 3\,|x - 1| < \epsilon,$$

si $|x - 1| < \epsilon / 3 = \delta$.



EJEMPLO 2. Demostrar :

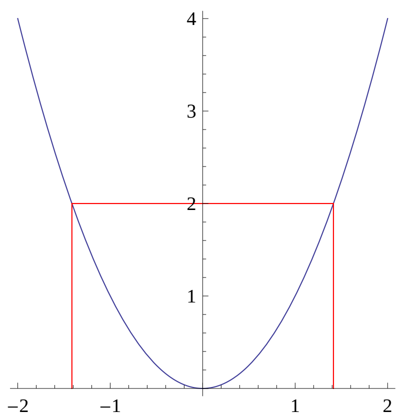
$$\lim_{x \to 0} g(x) = 0$$

$$g(x) = x^2$$

Sea $\epsilon > 0$, encontremos $\delta > 0$ tal que $\left| x^2 - 0 \right| < \epsilon$ si $0 < |x - 0| < \delta$.

$$\left| x^2 - 0 \right| = \left| x^2 \right| = x^2 = |x|^2 < \epsilon,$$

si $|x| < \sqrt{\epsilon} = \delta$.



EJEMPLO 3. Demostrar :

$$\lim_{x \to 1} g(x) = 1$$

$$g(x) = x^2$$

Sea $\epsilon > 0$, encontremos $\delta > 0$ tal que $\left| x^2 - 1 \right| < \epsilon$ si $0 < |x - 1| < \delta$.

$$\left| x^2 - 1 \right| = |(x + 1)(x - 1)| = |x + 1| \, |x - 1|$$

Si $|x - 1| < 1$, $-1 < x - 1 < 1$, $0 < x < 2$, $1 < x + 1 < 3$, $|x + 1| < 3$

Por lo tanto

$\left| x^2 - 1 \right| = |x + 1| \, |x - 1| < 3 \, |x - 1| < \epsilon$,

si $|x - 1| < \epsilon / 3$ y $|x - 1| < 1$

$\delta = \text{Min} \{1, \epsilon / 3\}$

2. LIMITES EN EL INFINITO

$$\lim_{x \to \infty} f(x) = L$$

Para todo $\epsilon > 0$ existe M > 0, tal que $|f(x) - L| < \epsilon$ si $x > M$

EJEMPLO. Demostrar:

$$\lim_{x \to \infty} f(x) = 0$$

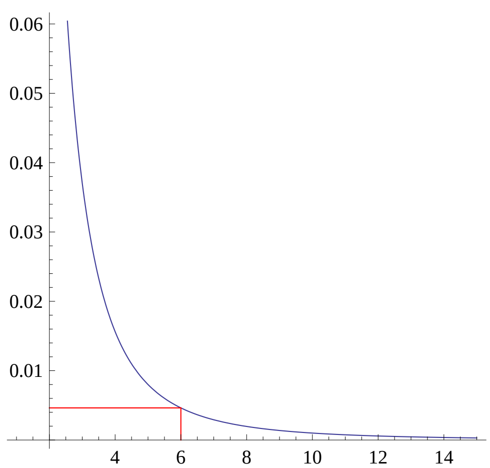
$$f(x) = \frac{1}{x^3}$$

Sea $\epsilon > 0$, encontremos $M > 0$ tal que $\left|\frac{1}{x^3} - 0\right| < \epsilon$ si $x > M$.

Si $x > 0$,

$$\left|\frac{1}{x^3} - 0\right| = \left|\frac{1}{x^3}\right| = \frac{1}{x^3} < \epsilon,$$

$$\text{si } \frac{1}{\epsilon} < x^3 \iff \sqrt[3]{\frac{1}{\epsilon}} < x. \text{ Por lo tanto } M = \sqrt[3]{\frac{1}{\epsilon}}$$



3. LIMITES INFINITOS

$$\lim_{x \to a} f(x) = \infty$$

Para todo M > 0 existe δ > 0, tal que f(x) > M si 0 < |x - a| < δ

EJEMPLO.

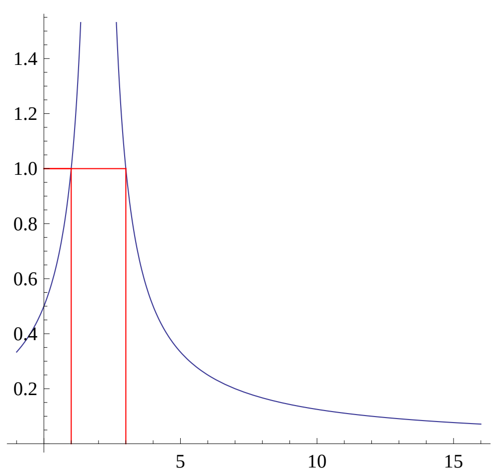
Demostrar : $\lim_{x \to 2} f(x) = \infty$

$$f(x) = \left| \frac{1}{x-2} \right|$$

$$f(x) = \left| \frac{1}{x-2} \right|$$

Sea M > 0, encontremos $\delta > 0$ tal que $\left| \frac{1}{x-2} \right| > M$ si $0 < |x-2| < \delta$

$$\left| \frac{1}{x-2} \right| > M \Leftrightarrow \frac{1}{|x-2|} < M \Leftrightarrow \frac{1}{M} < |x-2|, \ \delta = \frac{1}{M}$$

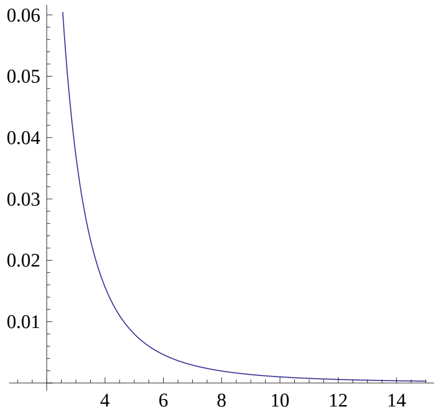


Herramientas

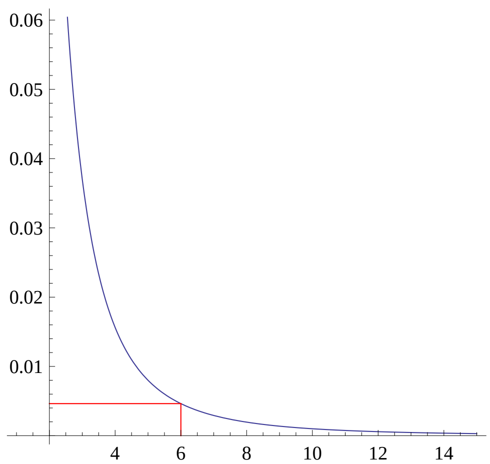
```
f[x_] := 1 / (x^3)

G1 = Graphics[{Red, Line[{{6, 0}, {6, f[6]}}]}];
G2 = Graphics[{Red, Line[{{6, f[6]}, {2, f[6]}}]}];
```

```
Show[G1, G2]
```

```
Plot[f[x], {x, 1, 15}, AspectRatio → 1 / 1.1]
```



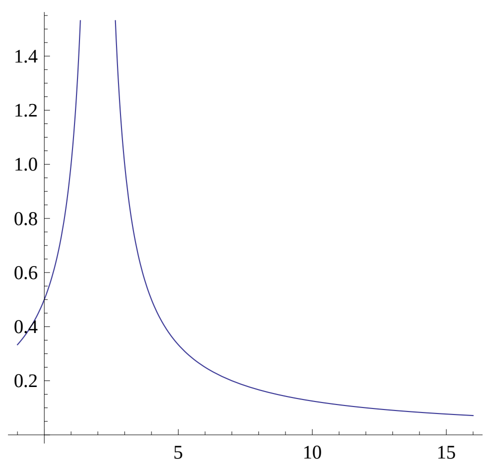

```
Show[%, %%]
```



In[19]:=
```
f[x_] := Abs[1 / (x - 2)]
```

In[33]:=
```
Plot[f[x], {x, -1, 16}, AspectRatio → 1 / 1.1]
```

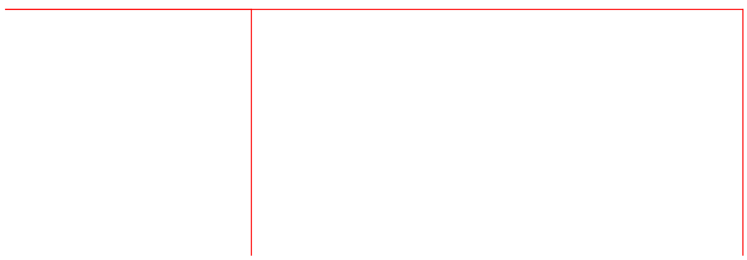
Out[33]=



In[28]:=
```
G1 = Graphics[{Red, Line[{{1, 0}, {1, f[1]}}]}];
G2 = Graphics[{Red, Line[{{1, f[1]}, {0, f[1]}}]}];
G3 = Graphics[{Red, Line[{{3, 0}, {3, f[3]}}]}];
G4 = Graphics[{Red, Line[{{3, f[3]}, {0, f[3]}}]}];
```

In[32]:=
```
Show[G1, G2, G3, G4]
```

Out[32]=



In[34]:=
```
Show[%, %%]
```

Out[34]=